Coherence – The waves maintaining constant and predictable phase difference are called coherent waves. Coherence requires that there is a connection between the amplitude and phase of the light at one point and time, and the amplitude and phase of the light at another point and time. If there are many waves of different frequencies, then the waves do not interfere and are said to be incoherent. It can be said that interference is a sure test of coherency.

When complete coherence is not achieved but some interference still exists then it refers to a condition of partial coherence.

Two types of coherence –

Temporal coherence – Temporal coherence (also called as longitudinal coherence or time coherence) is associated with the frequency spread of a wave pulse or difference in frequencies of waves. This type of coherence refers to the correlation between the wave field at a point and the field at the same point at a later time. If the phase difference measured at a single point in space at beginning and end of a time interval \( \Delta t \) remain predictable and constant, then waves are said to have temporal coherence in the time interval. The maximum value of the time interval in which waves maintain constant and have predictable phase difference is known as coherent time \( t_c \). Greater is \( t_c \), higher are the waves temporally coherent.

The beam is said to possess temporal coherence if the phase difference of the waves crossing P and Q at any instant remains constant

The temporal coherence also refers to the relative phase or coherence of waves at two separate locations along the direction of propagation of the waves. The maximum separation between two points along the direction of propagation of waves which maintain constant phase difference is known as temporal coherence length \( l_c \).

If coherent time is \( t_c \), coherence length \( l_c \) and bandwidth \( \Delta \omega \), then

\[
\Delta \omega = \frac{2\pi}{t_c}
\]

If \( c \) is velocity of light, then

\[
l_c = ct_c
\]

Since \( \Delta \omega = 2\pi \Delta \nu \)
Therefore, \[2\pi\Delta v = \frac{2\pi}{c}\]

Or \[\Delta v = \frac{1}{c}\]

Also, \[v = \frac{c}{\lambda}\]

Therefore, \[dv = \frac{c}{\lambda^2}d\lambda\]

Therefore \[dv = \frac{c}{l_c}\]

Comparing the above two equations, \[d\lambda = \frac{\lambda^2}{l_c}\]

Here, \(d\lambda\) is called the natural line width. Hence, temporal coherence depends upon the value of coherent length and coherent time.

**Spatial coherence** – Light is said to possess spatial coherence if the phase difference of the waves crossing the two points lying on a plane perpendicular to the direction of propagation of the beam is time-independent. This type of coherence is also called as transverse or lateral coherence. It is measure of minimum separation across the wave from where two waves remain coherent.

Spatial coherence describes the ability for two points in space, in the extent of a wave to interfere, when averaged over time. More precisely, the spatial coherence is the cross correlation between two points in a wave for all times. If a wave has only one value of amplitude over an infinite length, it is perfectly spatially coherent.

In the figure S is a source and S_1 and S_2 are two slits of separation \(l\). If interference occurs for \(l_{\text{max}} = l_o\) then this dimension is called the spatial coherence length of the source.
The condition for coherence of $S_1$ and $S_2$ is

$$\frac{\lambda}{a} > \frac{l}{D} \quad \text{or} \quad l < \frac{\lambda}{a/D}$$

Since, $\frac{a}{D} = \theta$, angle subtended by width of slit $S$. Thus, we get

$$l_\omega = \frac{\lambda}{\theta}$$

**Q factor for light** – The monochromaticity can be expressed in terms of spectral purity factor $Q$ defined as

$$Q = \frac{\lambda}{d\lambda} = \frac{\lambda}{\Delta\lambda} = \frac{\nu}{\Delta\nu}$$

For absolutely monochromatic light $\Delta\nu = 0$ and the purity factor is infinite which is practically impossible. For partially coherent light, coherence length $l_c$ is given by

$$l_c = \frac{\lambda}{\Delta\lambda} = Q\lambda$$

**Visibility as a measure of coherence:**

Absolute coherence is impossible and therefore, one should talk of partial coherence or rather degree of coherence.
We know that interference takes place when waves are coherent. Consider two wave trains of light, each of length $l_c$, overlapping to their full extent. If this is so, interference pattern will have distinct maxima and minima of the highest degree of contrast. But when wave trains overlap only partially, there will be interference but degree of contrast is less depending on amount of overlap. If the wave trains do not overlap, there will be no interference. Thus, contrast or visibility is a measure of degree of coherence.

Visibility, $V$ is defined as the ratio of difference between maximum intensity $I_{\text{max}}$ and minimum intensity $I_{\text{min}}$ to the sum of these intensities.

$$V = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}$$
Generally, contrast or visibility of 0.8 is considered high, and at 0.2 fringes are barely visible, that is, visibility must be related to degree of coherence.

Consider two interfering waves each of intensity $I_o$. Each of them consists of coherent part say $I_c$ and incoherent part $I_{inc}$. If $C$ be the degree of coherence, then

$$I_c = CI_o \quad \text{and} \quad I_{inc} = 1 - C \cdot I_o$$

When superimposed, the coherent part shall interfere adding there by their amplitudes whereas for the incoherent parts the intensities are simply added. Thus,

$$I_{max} = 4CI_o + 2 \cdot 1 - C \cdot I_o$$

$$I_{min} = 0 + 2 \cdot 1 - C \cdot I_o$$

Therefore visibility,

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{4CI_o + 2 \cdot 1 - C \cdot I_o - 2 \cdot 1 - C \cdot I_o}{4CI_o + 2 \cdot 1 - C \cdot I_o + 2 \cdot 1 - C \cdot I_o}$$

Or

$$V = \frac{4CI_o}{4I_o} = C$$

Thus, degree of contrast is a measure of degree of coherence between waves of equal intensities.

If the intensities of waves are unequal, the visibility is given by

$$V = \frac{2\sqrt{I_1I_2}}{I_1 + I_2} \cdot C$$

Visibility in interference measures degree of coherence but the two, interference and coherence, should not be mistaken for one. Thus, coherence is the property of light and interference is the process of interaction whose result depends on the above property.
LASER

Einstein’s coefficients –

Any emission and absorption process requires at least two energy states or levels, say \( E_1 \) and \( E_2 \) such that \( E_2 > E_1 \). Consider this system is put in a radiation field of photons of frequency \( v \), and \( v = \frac{E_2 - E_1}{h} \). Here \( h \) is Planck’s constant.

The system may interact the radiation field in three different ways.

**Absorption** – The system, for example, an atom may absorb a photon and raise the system from state \( E_1 \) to state \( E_2 \). This process is absorption of a quantum of light. If \( N_1 \) be the number of atoms per unit volume in the collection of energy \( E_1 \), then the absorption rate of transition shall be proportional to both \( N_1 \), and the number of photons available per unit volume \( u \) at correct frequency \( v \). The absorption rate is given by

\[
P_{12} = N_1 B_{12} u, \quad \ldots 1
\]

Where \( B_{12} \) is constant of proportionality measuring transition probability per unit time per unit photons density leading to absorption. This probability of transition leading to absorption depends on \( E_1 \) and \( E_2 \).

**Spontaneous emission** – An atom lying in higher state \( E_2 \) cannot remain in excited state indefinitely, it transits to lower state spontaneously by emitting a photon of frequency, \( v = \frac{E_2 - E_1}{h} \). However, this process does not in any way require the presence of photon field and hence, the emitted photons have random direction of momentum, random phase and polarization. Also, it has less degree of monochromaticity. The rate of spontaneous emission is proportional to the population of the excited state only and is equal to

\[
N_2 A_{21} \quad \ldots 2
\]

Where \( A_{21} \) is constant of proportionality measuring probability per unit time of spontaneous emission.

**Stimulated emission** – According to Einstein, the photon field of proper energy and phase may trigger an induced transition from state \( E_2 \) to state \( E_1 \), emitting a photon. Both incident or inducing photon and induced photons have the same
energy, identical momentum (both magnitude and direction) and identical polarization as per Bose-Einstein law.

The rate of stimulated emission depends on both the photon density, \( u_v \) and the population in the excited state \( N_2 \), and is equal to

\[
N_2 B_{21} u_v \quad \text{...3}
\]

Where \( B_{21} \) is constant of proportionality and measures the probability per unit time per unit photons density of induced or stimulated emission.

Constants \( B_{12} \), \( A_{21} \) and \( B_{21} \) are called Einstein’s transition coefficients.

It is clear that in absorption and stimulated emission, photon field or incident photon plays a vital role. Induced absorption occurs when incident photon and atomic oscillator (excited atom is equivalent to atom in ground state plus a photon) has opposite phase and stimulated emission occurs when these are in the same phase.

When atomic system is in thermal equilibrium with the radiation field, then rate of upward transition is equal to rate of downward transition, that is,

\[
N_1 B_{12} u_v = N_2 A_{21} + N_2 B_{21} u_v \quad \text{...4}
\]

Solving for \( u_v \),

\[
u_v \quad \text{...5}
\]

The population density \( N \) in a given state depends on the energy \( E \) of the state and temperature \( T \) given by Boltzmann’s equation

\[
N = N_0 e^{-E/kT} \quad \text{...6}
\]

The relative population density is therefore given by

\[
\frac{N_1}{N_2} = \frac{e^{-E_1/kT}}{e^{-E_2/kT}} = e^{(E_2 - E_1)/kT} = e^{h\nu/kT} \quad \text{...7}
\]

Substituting the value of \( \frac{N_1}{N_2} \) from eq...7 in eq...5, we get
Einstein proved thermodynamically that the probability of stimulated (induced) absorption is equal to the probability of stimulated (induced) emission, that is,

\[ B_{12} = B_{21} \]  …10

Therefore, eq…9 reduces to

\[ u_v = \frac{A_{21}}{B_{21} e^{\frac{hv}{kT}} - 1} \]  \[ \frac{1}{B_{21} e^{\frac{hv}{kT}} - 1} \]  …11

From Planck's radiation law we know that radiation field density: number of photons per unit volume in the frequency range \( \nu \) and \( \nu + d\nu \) is given by

\[ u_v = \frac{8\pi h\nu^3}{c^3} \left( \frac{1}{e^{\frac{hv}{kT}} - 1} \right) \]  …12

Comparing eq…10 and eq…11, we get

\[ \frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3} \]  …13

Equations 10 and 13 are called Einstein's relations. From these results

1. Stimulated emission and absorption are competing processes.
2. The rate of spontaneous emission is far larger than stimulated emission.
3. The ratio of \( A_{21} \) and \( B_{21} \) is proportional to cube of frequency of the transition.
   This means it shall be more and more difficult to have lasers of higher frequencies, particularly, X-rays and Gamma-rays.

To have laser action, the process of stimulated emission should dominate the other two competitive processes.

**34. Discuss the chief characteristics of laser.**
The word LASER stands for Light Amplification by Stimulated Emission of Radiation. Laser is a light source very much different from traditional light sources like candle or electric bulb, which are basically used for illumination purposes.

The chief characteristics of laser are:

**Directionality** – The conventional light sources emit light in all directions. In case of laser, active material is in a cylindrical resonant cavity. Any light that is traveling in a direction other than parallel to the cavity axis is eliminated and only light that is traveling parallel to the axis emerges from the cavity and becomes the laser beam. Hence, the light emitted by a laser is only in one direction. The directionality of a laser beam is expressed in terms of beam divergence. Light from a laser diverges very little. Up to certain distance, beam remains a bundle of parallel light rays. The distance from the laser over which the light rays remain parallel is known as Rayleigh range. The laser beam diverges beyond Rayleigh range.

The beam divergence due to size of the beam waist $d_\omega$ and wavelength $\lambda$ is given by

$$\theta = \frac{2\lambda}{\pi d_\omega}$$

The beam divergence due to diffraction is determined from Rayleigh's criterion and is given by

$$\theta = \frac{1.22\lambda}{D} \quad \text{Where } \lambda \text{ is wavelength and } D \text{ is diameter of laser's aperture.}$$

From the above two formulae, we see that beam divergence is inversely proportional to $d_\omega$ or $D$. 
**Intensity** – The power output of laser may vary from a few mW to a few kW. But this energy is concentrated in a beam of very small cross-sectional area. Hence, intensity of a laser beam is very high.

The intensity of a laser beam is given by

\[ \text{Intensity} = \frac{\text{Power}}{\text{Area}} \quad \text{Unit is watt per square meter} \]

\[ I = \frac{P}{A} \]

The intensity is also given by (approximately)

\[ I = \left( \frac{10}{\lambda} \right)^2 P \]

**Coherence** – Light waves are said to be coherent if they are in phase with each other. The light that emerges from conventional light sources are incoherent but light from a laser is a resultant of a large number of identical photons, which are in phase and therefore exhibits a high degree of coherence. Thus, we get a highly coherent beam of light from a laser source.

**Monochromaticity** – Light containing single wavelength or frequency of oscillation is said to be monochromatic and the source a monochromatic source. In actual practice, light from any source consists of a band of frequencies, \( \Delta v \), closely spaced around the central frequency \( v_0 \). The band of frequencies \( \Delta v \) is called the line-width or bandwidth. The light from conventional sources has large bandwidths of the order of \( 10^{10} \) Hz or more. On the other hand, light from a laser, is highly monochromatic having bandwidth of the order of 100 Hz or so.

Thus, laser is a highly directional, energetic, coherent and monochromatic beam of light, far different from ordinary light.

**Population inversion** – Under ordinary conditions of thermal equilibrium, the number of atoms in higher energy level is considerably smaller than the number of atoms in lower energy level, so that there is very little stimulated emission compared to absorption. In such situation, incident photon is more likely to be absorbed rather than emission. Hence, laser action will not take place. If however, the larger numbers of atoms are made available in the higher energy level as compared to lower energy level, then, stimulated emission will take place easily. This process of achieving the larger number of atoms in the higher energy level than the lower energy level is known as population inversion. This means
that population inversion is a necessary condition to be satisfied for causing the amplification of light.

**Pumping** – In order to realize and maintain the state of population inversion, it is necessary that the atoms must be continuously promoted from the lower level to the excited level. Energy is to be supplied somehow to the laser medium to raise atoms from the lower level to the excited level and for maintaining population at the excited level at a value greater than that of the lower energy. The process by which atoms are raised from the lower level to the upper level is called pumping. Population inversion cannot be achieved by heating the material. The most commonly used pumping methods are: optical pumping, electric discharge, direct conversion, chemical reaction, inelastic atom-atom collision etc.

**Meta-stable states** – An atom can be excited to a higher energy level by supplying energy to it. Normally, excited states have short life times and release their excess energy in a matter of nano-seconds ($10^{-9}$ second) by spontaneous emission. Thus, population inversion cannot be established. To achieve this, an excited state with a longer life time is needed. Such states where atoms remain for an appreciable time ($10^{-6}$ to $10^{-3}$ second) are known as meta-stable states. In meta-stable state, population of atoms can exceed the population of atoms of a lower level and lead to the state of population inversion. If the meta-stable states do not exist, there could be no population inversion, no stimulated emission and hence no laser action.

**The essential components of a laser are** –

**Active medium** – Atoms in general are characterized by a large number of energy levels. However, all types of atoms are not suitable for laser operation. Even in a medium consisting of different species of atoms, only a fraction of atoms of a particular species are suitable for stimulated emission and laser action. Those atoms which cause light amplification are called active centers. The rest of the medium acts as host and supports active centers. The medium hosting the active centers is called an active medium. An active medium is thus a medium which, when excited, reaches the state of population inversion, and
eventually causes light amplification. An active medium must have at least one meta-stable state. The active medium may be a solid, a liquid or a gas, on the basis of which different lasers are classified.

**Nd:YAG solid-state laser**

**Optical resonator** – Optical resonator is required to provide high photon field. It consists of two mirrors facing each other. The active medium is enclosed by this cavity. One of the mirrors is fully reflective while the other is partially reflective, say 99.9%. The optical cavity so formed is made use of to make stimulated emission possible in more number of atoms in the active medium, which increases the intensity of the laser beam.

Initially, the active centers are in ground state. After pumping process, state of population inversion is achieved. Spontaneous photons are emitted in all directions in initial stage. To generate coherent light, photons traveling in a specific direction are selected while others are rejected. These particular photons are made to pass through the medium a number of times with the help of two mirrors, due to which more and more strength is achieved. Laser beam oscillations begin when the amount of amplified light becomes equal to the total amount of light lost through the sides of the resonator. As the oscillations build up to enough intensity, it emerges through the partially reflective (partially transmitting) mirror as highly collimated, coherent and intense beam of light called as laser light.

**Energy source** – Energy source raises the system to an excited state, causing population inversion via pumping that breaks the thermal equilibrium.

**Essential requirements for producing laser action & threshold condition for sustained laser action.**

In order to have laser action, three requirements must be satisfied.

1. Active medium of desired energy levels,
2. Optical resonator to provide high photon field and
3. Pumping input energy to yield population inversion so that stimulated 
(induced) emission dominates spontaneous emission.

When light beam bounces back and forth the parallel mirrors of the cavity, it is 
not only amplified via induced emission but there are losses also.

Most important of these losses are:

1. Transmission at mirrors; the output mirror is partially reflecting to give output.
2. Absorption and scattering by mirrors.
3. Absorption by laser medium due to undesired transitions.
4. Diffraction at mirrors due to their finite size.
5. Scattering at optical in-homogeneities particularly in solid laser.

For laser action the gain due to stimulated emission should be greater than or 
equal to these losses.

Let separation of mirrors in optical resonator be $d$ and the refractive index of 
cavity medium be $n_o$. The losses other than finite reflectivity may be grouped into 
a loss constant per unit length $\alpha$. Thus, when light bounces back and forth once, 
energy decreases by a factor of $e^{-2\alpha d}$ and increases by a factor $e^{2\beta d}$, where $\beta$ is 
gain constant. If $R_1$ and $R_2$ be the reflectivity of two mirrors, then fractional gain 
in energy will be

$$G = e^{2\beta d} e^{-2\alpha d} R_1 R_2$$  \(\ldots 1\)

For laser action to sustain, net gain $G$ should be greater than or equal to unity, 
that is,

$$G = e^{2\beta d} e^{-2\alpha d} R_1 R_2 \geq 1$$  \(\ldots 2\)

The threshold condition is therefore,

$$G = R_1 R_2 e^{2 \beta - \alpha d} = 1$$  \(\ldots 3\)

Or

$$2\beta d - 2\alpha d + \log_e R_1 R_2 \geq 1$$  \(\ldots 4\)

Or

$$\beta \geq \left[ \alpha - \frac{\log_e R_1 R_2}{2d} \right]$$  \(\ldots 5\)
Eq. 5 is threshold condition for sustained oscillations. The last term on the RHS gives the laser output.

**Semiconductor lasers** – are derived from the light emitting diodes (LED). LED is basically a highly doped p-n junction which quite small in size and compact. It converts electric energy directly into light. Because of small size the population inversion density becomes quite large making these lasers highly efficient. These lasers generally operate in continuous wave mode (cw-mode) but can be made to operate in pulsed mode also. Most of these lasers operate in infra-red region but these can be made to operate from ultra-violet to infra-red by selecting semi-conducting material or combination of materials.

![Diagram of p-n junction and Fermi level](image)

When heavily doped p-n junction diode is forward biased, electrons from n-region and holes from p-region drifts towards each other and when recombine across the band gap, excess energy is released in the form of light. When this junction diode is mounted in an optical resonator, the emission stimulates like other lasers and we have semi-conductor laser.

In a semi-conductor laser forward biased heavily doped p-n junction serves the active medium. Because of forward biasing, current readily flows due to reduced energy barrier and provides pumping leading to population inversion which is achieved because of high carrier density in the conduction band, compared with the valence band.

![Diagram of electron energy and wavenumber](image)
Without pumping, most of the electrons are in the valence band. A pump beam with photon energy somewhat above the band gap energy can excite electrons into a higher state in the conduction band, from where they quickly decay to states near the bottom of the conduction band. At the same time, the generated holes in the valence band move to the top of the valence band. Electrons in the conduction band can then recombine with these holes, emitting photons with energy near the band gap energy. This process can also be stimulated by incoming photons with suitable energy.

Most semiconductor lasers are laser diodes, which are pumped with an electrical current in a region where an n-doped and a p-doped semiconductor material meet. However, there are also optically pumped semiconductor lasers, where carriers are generated by absorbed pump light.

Common materials for semiconductor lasers devices are GaAs (gallium arsenide), AlGaAs (aluminum gallium arsenide), GaP (gallium phosphide), InGaP (indium gallium phosphide), GaN (gallium nitride), InGaAs (indium gallium arsenide), InP (indium phosphide) and GaInP (gallium indium phosphide)

These are all direct band gap semiconductors, as indirect band gap semiconductors such as silicon do not exhibit significant light emission. By choosing material compositions with different band gaps, one can achieve different emission wavelengths. The emission wavelength is typically just above the band gap wavelength. While the most common semiconductor lasers are operating in the near-infrared spectral regions, some others (often based on gallium nitrides) generate blue or violet light, while quantum cascade lasers can emit at wavelengths beyond 10 μm.

Applications of semiconductor lasers are extremely widespread, including as diverse areas as optical data transmission, optical data storage, metrology, spectroscopy, material processing, pumping solid state lasers (diode-pumped lasers), and various kinds of medical treatments.

**Q switching** - is a technique to obtain energetic short (but not ultra short) pulses from a laser by modulating the intra cavity losses and thus the Q factor of the laser resonator. The technique is mainly applied for the generation of nanosecond pulses of high energy and peak power with solid state bulk lasers.

Power is the rate of flow of energy. Normally power flowing per pulse in lasers range from 10J to 300J which is not very high. But if pulse lasts for 0.5ms, 10J per pulse has power as large as 20kW. Some lasers produces 300J in 5ns resulting in very high power as large as 60GW. Thus higher powers may be produced by reducing pulse duration. This is done by the Q-switching.
By rotating mirror - One of the methods of reducing pulse duration is by using a rotating mirror at one end of the laser resonator, while keeping the semi-transparent mirror at the other end at place. Under this situation the system is pumped to population inversion but laser action does occur till rotating mirror comes in position to form the optical resonator. At that instant, all the accumulated energy is dumped into one single very short giant pulse.

By optical switch – The mechanical system of rotating mirror can be replaced by an optical switch. Electro-optic crystal placed between two crossed polarizers arranged at 45° at characteristic axis, is placed co-axially in the optical resonator near fully reflecting mirror. Till crystal has no bias, no light comes out of crossed polarizers and there is no laser action. When d.c. voltage of correct amplitude is applied, birefringence introduces a phase of $\pi/2$ leading to transmission of light and hence, laser action. The action is continued till the non-linear crystal has biasing, that is, d.c. voltage pulse continues.
Mode Locking - is a method (or actually a group of methods) to obtain ultra short pulses from lasers, which are then called mode-locked lasers. Here, the laser resonator contains either an active element (an optical modulator) or a nonlinear passive element which leads to the formation of an ultra short pulse circulating in the laser resonator. In the steady state, the various effects influencing the circulating pulse are in a balance so that the pulse parameters are unchanged after each completed round trip, or often even nearly constant throughout each round trip. Each time the pulse hits the output coupler mirror, a usable pulse is emitted, so that a regular pulse train leaves the laser. Assuming a single circulating pulse, the pulse repetition period corresponds to the resonator round-trip time (typically several nanoseconds), while the pulse duration is much lower: typically between 30 fs and 30 ps, in extreme cases down to ≈5 fs. For that reason, the peak power of a mode-locked laser can be orders of magnitude higher than the average power.
Comparison of non-mode locked and mode-locked lasers (d) In non-mode locked lasers phases are random and instantaneous power seldom exceeds average power. (e) Cosine waves add in phase, all the three waves have zero phase at t = 0. Phase locked laser pulses are produced with separation $2\frac{\ln}{c}$. 